

# Electrical control of the Kondo effect in a helical edge liquid

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Magnetic impurities affect the transport properties of the helical edge states of quantum spin Hall insulators by causing single-electron backscattering. We study such a system in the presence of a Rashba spin-orbit interaction induced by an external electric field, showing that this can be used to control the Kondo temperature, as well as the correction to the dc conductance due to the impurity.

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*Introduction.* The discovery that HgTe quantum wells support a quantum spin Hall (QSH) state [1] has set off an avalanche of studies addressing the properties of this novel phase of matter [2]. A key issue has been to determine the conditions for stability of the current-carrying states at the edge of the sample as this is the feature that most directly impacts prospects for future applications in electronics/spintronics. In the simplest picture of a QSH system the edge states are *helical*, with counter-propagating electrons carrying opposite spins. By time-reversal invariance electron transport then becomes ballistic, provided that the electron-electron (e-e) interaction is sufficiently well-screened so that higher-order scattering processes do not come into play [3, 4].

The picture gets an added twist when including effects from magnetic impurities, contributed by dopant ions or electrons trapped by potential inhomogeneities. Since an edge electron can backscatter from an impurity via spin exchange, time-reversal invariance no longer protects the helical states from mixing. In addition, correlated two-electron [5] and inelastic single-electron processes [6, 7] must now also be accounted for. As a result, at high temperatures  $T$  electron scattering off the impurity leads to a  $\ln(T)$  correction of the conductance at low frequencies  $\omega$  [8], which, however, vanishes in the dc limit  $\omega \rightarrow 0$  [9]. At low  $T$ , for weak e-e interactions, the quantized edge conductance  $G_0 = e^2/h$  is restored as  $T \rightarrow 0$  with power laws distinctive of a helical edge liquid. For strong interactions the edge liquid freezes into an insulator at  $T = 0$ , with thermally induced transport via tunneling of fractionalized charge excitations through the impurity [8].

A more complete description of edge transport in a QSH system must include also the presence of a Rashba spin-orbit interaction. This interaction, which can be tuned by an external gate voltage, is a built-in feature of a quantum well [10]. In fact, HgTe quantum wells exhibit some of the largest known Rashba couplings of any semiconductor heterostructures [11]. As a consequence, spin is no longer conserved, contrary to what is assumed in the minimal model of a QSH system [12]. However, since the Rashba interaction preserves time-reversal invariance, Kramers' theorem guarantees that the edge states are still connected via a time-reversal

transformation ("Kramers pair") [13]. Provided that the Rashba interaction is spatially uniform and the e-e interaction is not too strong, this ensures the robustness of the helical edge liquid [14].

What is the physics with both Kondo *and* Rashba interactions present? In this paper we address this question with a renormalization group (RG) analysis as well as a linear-response and rate-equation approach. Specifically, we predict that the Kondo temperature  $T_K$  – which sets the scale below which the electrons screen the impurity – can be controlled by varying the strength of the Rashba interaction. Surprisingly, for a strongly anisotropic Kondo exchange, a non-collinear spin interaction mediated by the Rashba coupling becomes relevant (in the sense of RG) and competes with the Kondo screening. This challenges the expectation that the Kondo effect is stable against time-reversal invariant perturbations [15]. Moreover, we show that the impurity contribution to the dc conductance at temperatures  $T > T_K$  can be switched on and off by adjusting the Rashba coupling. With the Rashba coupling being tunable by a gate voltage, this suggests a new inroad to control charge transport at the edge of a QSH device.

*Model.* To model the edge electrons, we introduce the two-spinors  $\Psi^T = (\psi_\uparrow, \psi_\downarrow)$ , where  $\psi_\uparrow$  ( $\psi_\downarrow$ ) annihilates a right-moving (left-moving) electron with spin-up (spin-down) along the growth direction of the quantum well. Neglecting e-e interactions, the edge Hamiltonian can then be written as

$$\begin{aligned} H = & v_F \int dx \Psi^\dagger(x) [-i\sigma^z \partial_x] \Psi(x) + \\ & + \alpha \int dx \Psi^\dagger(x) [-i\sigma^y \partial_x] \Psi(x) + \\ & + \Psi^\dagger(0) [J_x \sigma^x S^x + J_y \sigma^y S^y + J_z \sigma^z S^z] \Psi(0), \end{aligned} \quad (1)$$

with  $v_F$  the Fermi velocity parameterizing the linear kinetic energy. The second term encodes the Rashba interaction of strength  $\alpha$ , with the third term being an anti-ferromagnetic Kondo interaction between electrons (with Pauli matrices  $\sigma^i$ ,  $i = x, y, z$ ) and a spin-1/2 magnetic impurity (with Pauli matrices  $S^i$ ,  $i = x, y, z$ ) at  $x = 0$ . The spin-orbit induced magnetic anisotropy for an impurity at a quantum well interface [16] implies that  $J_x = J_y \neq J_z$

[17]. Unless otherwise stated, we use  $\hbar = k_B \equiv 1$ .

The Rashba term in Eq. (1) can be absorbed into the kinetic term by a spinor rotation  $\Psi' = e^{-i\sigma^x\theta/2}\Psi$  [18]. By rotating also the impurity spin,  $\mathbf{S}' = e^{-iS^x\theta/2}\mathbf{S}e^{iS^x\theta/2}$ , one obtains  $H = H'_0 + H'_K$ , with

$$H'_0 = v_\alpha \int dx \Psi'^{\dagger}(x) \left[ -i\sigma^{z'} \partial_x \right] \Psi'(x) \quad (2)$$

$$H'_K = \Psi'^{\dagger}(0) [J_x \sigma^x S^x + J'_y \sigma^{y'} S^{y'} + J'_z \sigma^{z'} S^{z'} + J_E (\sigma^{y'} S^{z'} + \sigma^{z'} S^{y'})] \Psi'(0), \quad (3)$$

where  $J'_y = J_y \cos^2 \theta + J_z \sin^2 \theta$ ,  $J'_z = J_z \cos^2 \theta + J_y \sin^2 \theta$  and  $J_E = (J_y - J_z) \cos \theta \sin \theta$ . The Rashba rotation angle  $\theta$  is determined through  $\cos \theta = v_F/v_\alpha$ ,  $\sin \theta = \alpha/v_\alpha$  and  $v_\alpha = \sqrt{v_F^2 + \alpha^2}$ . Note that the spin in the rotated basis is quantized along the  $z'$ -direction which forms an angle  $\theta$  with the  $z$ -axis. Also note that the Kondo interaction in the new basis not only becomes spin-nonconserving, but also picks up a non-collinear term for  $J_y \neq J_z$ .

Including e-e interactions, and assuming a band filling incommensurate with the lattice [2], time-reversal invariance constrains the possible scattering channels in the rotated basis to dispersive ( $\sim g_d$ ) and forward ( $\sim g_f$ ) scattering, in addition to correlated two-particle backscattering ( $\sim g_{bs}$ ) [3, 4] and inelastic single-particle backscattering ( $\sim g_{ie}$ ) [6, 7, 19] at the impurity site. Adding the corresponding interaction terms to  $H'_0$  and  $H'_K$  in (2) and (3), and employing standard bosonization [20], the full Hamiltonian for the edge liquid can now be expressed as a free boson model,  $(v/2) \int dx ((\partial_x \varphi)^2 + (\partial_x \vartheta)^2)$ , with three local terms added at  $x = 0$ :

$$H'_K = \frac{A}{\kappa} \cos(\sqrt{4\pi K} \varphi) + \frac{B}{\kappa} \sin(\sqrt{4\pi K} \varphi) + \frac{C}{\sqrt{K}} \partial_x \vartheta \quad (4)$$

$$H'_{bs} = \frac{g_{bs}}{2(\pi\kappa)^2} \cos(\sqrt{16\pi K} \varphi) \quad (5)$$

$$H'_{ie} = \frac{g_{ie}}{2\pi^2 \sqrt{K}} : (\partial_x^2 \vartheta) \cos(\sqrt{4\pi K} \varphi) : \quad (6)$$

Here  $\varphi$  is a nonchiral Bose field with  $\vartheta$  its dual,  $v\partial_x \vartheta = \partial_t \varphi$  with  $v = [(v_\alpha + g_f/\pi)^2 - (g_d/\pi)^2]^{1/2}$ ,  $K = [(\pi v_\alpha + g_f - g_d)/(\pi v_\alpha + g_f + g_d)]^{1/2}$  and  $\kappa \approx v_F/D$  is the edge state penetration depth acting as short-distance cutoff with  $D$  the band width, and  $: \dots :$  denotes normal ordering. In  $H'_K$  we have defined  $A = J_x S^x/\pi$ ,  $B = (J'_y S^{y'} + J_E S^{z'})/\pi$  and  $C = (J'_z S^{z'} + J_E S^{y'})/\pi$ . The presence of  $H'_{ie}$  requires breaking of the  $U(1)$  spin symmetry, as brought about by the Rashba interaction.

*Kondo temperature.* The bosonized theory is tailor-made for a perturbative RG analysis, allowing us to determine the temperature scale below which the edge electrons couple strongly to the impurity. We first note that the backscattering term in (5) is that of the well-known boundary-sine Gordon model. For  $K < 2/3$  it dominates over the inelastic backscattering in (6), and turns relevant for  $K < 1/4$  with a weak to strong-coupling crossover at

a temperature  $T_{bs} \approx D g_{bs}^{1/(1-4K)}$  [21]. As a consequence, the enhancement of backscattering as  $T \rightarrow 0$  results in a zero-temperature insulating state when  $K < 1/4$ .

Turning to the Rashba-rotated Kondo interaction  $H'_K$  in Eq. (4), we obtain for its one-loop RG equations:

$$\begin{aligned} \partial_l \tilde{J}_x &= (1-K) \tilde{J}_x + \nu K (\tilde{J}'_y \tilde{J}'_z - \tilde{J}_{E1} \tilde{J}_{E2}) \\ \partial_l \tilde{J}'_y &= (1-K) \tilde{J}'_y + \nu K \tilde{J}_x \tilde{J}'_z, \quad \partial_l \tilde{J}'_z = \nu K \tilde{J}_x \tilde{J}'_y, \quad (7) \\ \partial_l \tilde{J}_{E1} &= (1-K) \tilde{J}_{E1} - \nu K \tilde{J}_x \tilde{J}_{E2}, \quad \partial_l \tilde{J}_{E2} = -\nu K \tilde{J}_x \tilde{J}_{E1}, \end{aligned}$$

with the "tilde" indicating that the couplings depend on the renormalization length  $l$ , and where  $\nu \equiv a_0/(\pi v)$  with  $a_0$  the lattice constant. The two terms proportional to  $J_E$  in Eq. (4) flow individually under RG, with the corresponding renormalized coupling constants here denoted  $\tilde{J}_{E1}$  and  $\tilde{J}_{E2}$ . In deriving Eqs. (7) we have used that higher-order contributions involving an intermediate process governed by  $H'_{bs}$  or  $H'_{ie}$  are suppressed, since these transfer spin or energy incompatible with  $H'_K$ . In a recent work [22], Kondo scattering without Rashba interaction was studied, and different physics in the regime  $\nu J_z \geq 2K$  was found, not accessible perturbatively in  $\nu J_z$ . Since its realization in an HgTe quantum well requires anomalously weak screening of the e-e interaction we do not consider this regime here.

The role of the Rashba rotation in Eqs. (7) is both to determine the bare values  $\tilde{J}'_{y,z}(l=0) \equiv J'_{y,z}$  and to introduce the non-collinear couplings  $\tilde{J}_{E1,E2}$ . To explore the outcome, we first examine the case of a strongly screened e-e interaction,  $K \approx 1$ . For this case, the first-order terms of Eq. (7) can be neglected and  $\tilde{J}_{E1} = \tilde{J}_{E2} = \tilde{J}_E$ , since their scaling equations will be identical. In this limit,  $\tilde{J}_E$  quickly flows to zero. We take the Kondo temperature  $T_K$  to be the value of  $T = D \exp(-l)$  where one of the couplings in Eq. (7) first grows past  $1/(\nu K)$ , making the renormalized  $H'_K$  in Eq. (4) dominate the free theory. For  $K \approx 1$  we then see that

$$T_K \approx D \exp \left( -\frac{1}{\nu J_x} \frac{\text{arcsinh}(\zeta)}{\zeta} \right), \quad (8)$$

where  $\zeta = \sqrt{(J_z/J_x)^2 - 1}$  is an anisotropy parameter [8]. Here the  $\theta$  dependence lies predominantly in  $\nu$ . Note that Kondo temperatures modified by spin-orbit couplings, as in (8), or by spin-dependent hopping, have recently been proposed also for ordinary conduction electrons [23–27].

In the opposite limit of a strong e-e interaction, the second-order terms of the scaling equations can be neglected, as long as  $1-K \gg \tilde{J}\nu K$ , for all  $\tilde{J} = \tilde{J}_x, \tilde{J}'_y, \tilde{J}_{E1}$ . The scaling equations in this limit reduce to  $\partial_l \tilde{J} = (1-K) \tilde{J}$ , with solutions  $\tilde{J} = J e^{(1-K)l}$ . With  $l = \ln(D/T)$ , one can now use the  $\tilde{J} = 1/(\nu K)$  criterion to find the Kondo temperature

$$T_K \approx D (J_{\max} \nu K)^{1/(1-K)}, \quad (9)$$

where  $J_{\max} = \max[J_x, J'_y, J_E]$ .

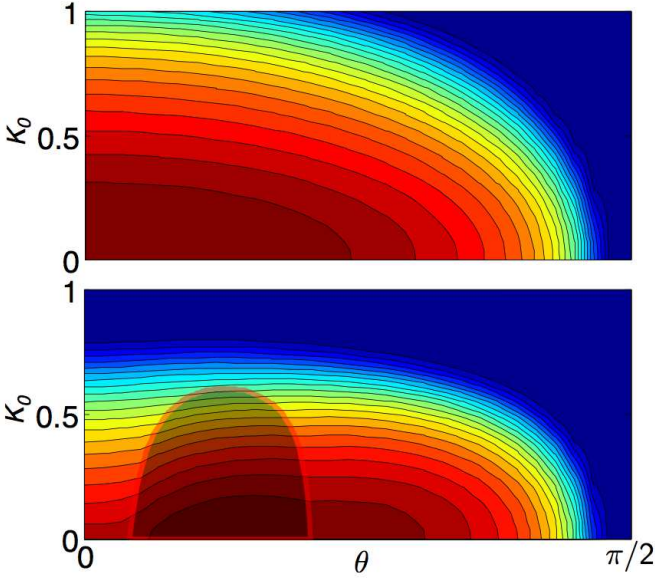


FIG. 1: The logarithm of the Kondo temperature,  $\log T_K$ , as a function of the Rashba angle  $\theta$  and the ordinary Luttinger parameter  $K_0$ . The  $\log T_K$  scale is arbitrary, with the difference between adjacent contour lines being  $\Delta(\log T_K) \approx 0.43$ . Red and blue color indicates high and low  $T_K$ , respectively. Top:  $J_x = J_y \geq J_z$ , bottom:  $J_x = J_y < J_z$  (here with  $J_x = J_y = J_z/10$ ). In the shaded area,  $\tilde{J}_{E1}$  dominates the perturbative RG flow, hence obstructing singlet formation.

In Fig. (1) we exhibit  $T_K$  for both "easy-plane" and "easy-axis" Kondo interaction. To isolate the effect of the Rashba interaction from that of the e-e interaction we choose to plot  $T_K$  as a function of  $\theta$  and  $K_0$ , with  $K_0 \equiv K(\theta=0)$  the ordinary Luttinger parameter. For  $|J_E| > |J_x|, |J'_y|$ , the non-collinear term  $\sim \sigma^{y'} S^{z'}$  in Eq. (3) dominates the RG flow for values of  $K$  in the shaded "dome" (the size of which is set by the ratio  $J_z/J_{x,y}$ ). As this term disfavors a spin singlet, Kondo screening will be obstructed in the corresponding interval of Rashba couplings. This runs contrary to the expectation that a spin-orbit interaction does not impair the Kondo effect [15, 28]. However, this expectation is rooted in a noninteracting quasiparticle picture which breaks down in one dimension. Instead a Luttinger liquid is formed, with strongly correlated electron scattering [20]. As suggested by our RG analysis, when this scattering gets enhanced with lower values of  $K$ , it boosts the effect of the non-collinear spin interaction that works against the Kondo screening.

*Conductance at low temperatures.* Away from the "dome" in Fig. (1), the Rashba-rotated Kondo interaction easily sustains a Kondo temperature  $T_K$  below which the impurity gets screened. When  $K > 1/4$  and two-particle backscattering is RG-irrelevant, there is no correction  $\delta G$  to the conductance at zero temperature: As explained by Maciejko *et al.* [8], the topological nature

of the QSH state implies a "healing" of the edge after the impurity has been effectively removed by the Kondo screening. For finite  $T \ll T_K$ , the leading correction  $\delta G$  is generated by either  $H'_{bs}$  or  $H'_{ie}$ , whatever operator has the lowest scaling dimension: For  $1/4 < K < 2/3$  ( $K > 2/3$ )  $H'_{bs}$  ( $H'_{ie}$ ) dominates, with  $\delta G_{bs} \sim (T/T_K)^{8K-2}$  [8] ( $\delta G_{ie} \sim (T/T_K)^{2K+2}$  [6, 7]). Thus, in the noninteracting limit  $K \approx 1$ , the presence of a Rashba interaction is revealed by a  $T^4$  scaling of the conductance, rather than the expected  $T^6$  scaling as in the minimal model with U(1) spin symmetry [8]. The picture changes dramatically for  $K < 1/4$ . Now  $H_{bs}$  turns RG-relevant, with  $g_{bs}$  entering a strong-coupling regime below the crossover-temperature  $T_{bs}$ , implying zero conductance at  $T = 0$ . At finite  $T \ll T_{bs}$ , instanton processes restore its finite value, with  $G \sim (T/T_{bs})^{2(1/4K-1)}$  [8]. To leading order this regime is blind to the Rashba interaction.

*Conductance and currents at high temperatures.* When  $T > \max(T_K, T_{bs})$ , scattering from  $H'_K$  as well as from  $H'_{bs}$  and  $H'_{ie}$  remain weak, and transport properties can be obtained perturbatively. We here focus on the correction  $\delta G$  to the conductance due to  $H'_K$ , noting that the contributions from  $H'_{bs}$  and  $H'_{ie}$  decouple and are insensitive to the strength of the Rashba interaction.

The current operator  $\hat{I}$  takes the form  $\hat{I} = (e/2) \partial_t (\cos \theta \Psi'^{\dagger} \sigma^{z'} \Psi' - \sin \theta \Psi'^{\dagger} \sigma^{y'} \Psi')$  in the rotated basis. After the unitary transformation  $U = e^{i\sqrt{\pi}\lambda\varphi(0)S^{z'}}$ , which removes the  $J'_z$ -term when  $\lambda = J'_z/\pi v\sqrt{K}$ , the bosonized part  $\delta \hat{I}$  of the current operator due to  $H'_K$  is

$$\delta \hat{I} = \frac{ie}{2\pi\kappa} \left[ \sum_{j=\pm} A_j e^{i\sqrt{\pi}(2\sqrt{K}-j\lambda)\varphi} S^j + iA_0 e^{i\sqrt{4\pi K}\varphi} S^z - \frac{iJ_x}{\sqrt{\pi K}} \sin \theta : \partial_x \vartheta e^{-i\sqrt{\pi}\lambda\varphi} : S^+ \right] + \text{h.c.} \quad (10)$$

where  $A_{\pm} = (1/2)(J_x \pm J_y) \cos \theta$ ,  $A_0 = (J_E/2) \cos \theta$ . Using the Kubo formula to calculate the conductance correction  $\delta G(\omega)$  at a frequency  $\omega$  in the limit  $J^2 \ll \omega \ll T$ , with  $J^2 = J_x^2, J_y^2, J_E^2$ , we then find

$$\delta G = -e^2 \sum_{j=-1}^{+1} F_j \cdot [A_j(T)]^2 - e^2 \mu \sin^2 \theta, \quad (11)$$

which, in this limit, is independent of  $\omega$ . Here  $A_j(T) = A_j \cdot (2\pi T/D)^{(\sqrt{K}-j\lambda/2)^2-1}$  to  $\mathcal{O}(J)$ . The constant  $\mu$  depends on  $J_x, \lambda$  and  $K$ , while  $F_j \equiv F(2\sqrt{K}-j\lambda)$  with  $F(x) = [\Gamma(x^2/4)]^2/[4\pi(\hbar v)^2\Gamma(x^2/2)]$ . At zero Rashba coupling,  $\theta = 0$ , Eq. (11) reproduces the finding in Refs. [8, 9]. By replacing the bare couplings by renormalized ones, the result in Eq. (11) can be RG-improved to numerically obtain  $\delta G$  to all orders in perturbation theory in a leading-log approximation. At  $\theta = 0$  this gives  $\delta G \sim \ln(T)$ , in agreement with Ref. 8.

As stressed in Ref. 9, the use of the Kubo formula rests on a perturbation expansion (in our case assuming that  $J^2 \ll \omega$ ) which breaks down as  $\omega \rightarrow 0$ . To

study the scaling of  $\delta G$  in the dc limit we will instead fall back on a semiclassical rate equation approximation where we assign classical probabilities to the states. This means that we restrict the current operator  $\hat{I}$  to its diagonal part,  $\propto \cos \theta$ . As indicated by the linear response calculations above, one expects this to be sufficient to capture the scaling behavior. A voltage  $V$  now adds a term  $H_V = \frac{eV}{2} \int dx (\psi_\uparrow^\dagger \psi_\uparrow - \psi_\downarrow^\dagger \psi_\downarrow)$  to the Hamiltonian, and after the Rashba rotation we retain only its diagonal part as we did for  $\hat{I}$ . This restriction allows us to assign the bias  $V' = V \cos \theta$  between spin-up ( $\uparrow'$ ) and spin-down ( $\downarrow'$ ) states in the rotated basis. The analysis can now be patterned upon that in Ref. 9, with the construction of a rate equation for the impurity spin,

$$\partial_t P_{\uparrow'} = (\gamma_+ + \gamma'_- + \tilde{\gamma}_0^E) P_{\downarrow'} - (\gamma_- + \gamma'_+ + \tilde{\gamma}_0^E) P_{\uparrow'}, \quad (12)$$

with the  $\gamma$ -parameters encoding the various voltage-dependent spin-flip rates implied by  $H'_K$  in (3),

$$\sigma^\mp S^\pm \rightarrow \gamma_\pm, \quad \sigma^\pm S^\pm \rightarrow \gamma'_\mp, \quad \sigma^\mp S^z \rightarrow \gamma_\pm^E, \quad \sigma^z S^\pm \rightarrow \tilde{\gamma}_0^E.$$

Here  $\gamma_\pm = \gamma_0 \Lambda_\pm$ ,  $\gamma'_\pm = \gamma'_0 \Lambda_\pm$ ,  $\gamma_\pm^E = \gamma_0^E \Lambda_\pm$ , with  $\Lambda_\pm \equiv (1 \pm (eV/2T) \cos \theta)$  for  $eV \ll T$ , where the rates  $\gamma_0$ ,  $\gamma'_0$ ,  $\gamma_0^E$  and  $\tilde{\gamma}_0^E$  are determined from linear response in the regime  $\gamma_0, \gamma'_0, \gamma_0^E, \tilde{\gamma}_0^E \ll \omega \ll T$ , as in (11). Semiclassically the operator  $\sigma^+$  has a probability  $\cos^4(\theta/2)$  of transforming a left-moving electron into a right-moving electron, and a probability  $\sin^4(\theta/2)$  of transforming a right-mover into a left-mover, and vice versa for  $\sigma^-$ . Since  $\cos^4(\theta/2) - \sin^4(\theta/2) = \cos \theta$ , the current in the original basis is given by  $\delta I = -e (\gamma_+ P_{\downarrow'} - \gamma'_- P_{\downarrow'} + \gamma'_+ P_{\uparrow'} - \gamma_- P_{\uparrow'} + \gamma_+^E/2 - \gamma_-^E/2) \cos \theta$ . In the dc limit, i.e.  $\omega \ll J^2 \ll T$ , the conductance correction then becomes

$$\delta G = -\frac{e^2 \cos^2 \theta}{2T} \left[ \frac{4\gamma_0 \gamma'_0 + (\gamma_0 + \gamma'_0)(\gamma_0^E + \tilde{\gamma}_0^E) + \tilde{\gamma}_0^E \gamma_0^E}{\gamma_0 + \gamma'_0 + \tilde{\gamma}_0^E} \right]$$

with  $\gamma_0 \sim (J_x + J_y)^2 T^{2(\sqrt{K}-\lambda/2)^2-1}$ ,  $\gamma'_0 \sim (J_x - J_y)^2 T^{2(\sqrt{K}+\lambda/2)^2-1}$ ,  $\gamma_0^E \sim J_E^2 T^{2K-1}$ , and  $\tilde{\gamma}_0^E \sim J_E^2 T$ , where, for brevity, we have omitted various  $K$ -dependent prefactors. Note that with  $J_x = J_y$ , the vanishing  $\delta G$  becomes non-zero when turning on the Rashba interaction by an electric field. This suggests a means to manipulate the edge current by varying the bias of an external gate.

To explore this possibility we have calculated the  $\delta I$ - $V$  characteristics by exploiting a "tunneling formalism" [29, 30], using the same restrictions on the current operator and voltage as above. When  $J^2 \ll \omega \ll T$ ,  $eV$  we find

$$\delta I \approx -e \sum_{j=-1}^{+1} \text{Im} \{ B(K_j + ieV'/2\pi T, K_j - ieV'/2\pi T) \times C_j (T/D)^{2K_j-1} \sin[\pi(K_j - ieV'/2\pi T)] / \cos(\pi K_j) \} \quad (13)$$

for  $\delta I \equiv I - G_0 V$ , with  $K_j \equiv (\sqrt{K} - j\lambda/2)^2$  and  $B$  the beta function. Here  $C_{\pm 1} = c_\pm (J_x \pm J_y)^2$  and  $C_0 = c_0 J_E^2$ , with  $c_{\pm 0}$  constants depending on  $K$ ,  $\lambda$  and  $\theta$ , cf Fig. (2).

*Experimental realization.* Given our result in Eq. (13), is the Rashba-dependence of  $\delta I$  large enough to be seen in an experiment? As a case study, let us consider an  $\text{Mn}^{2+}$  ion implanted close to the edge of an HgTe quantum well [31].  $\text{Mn}^{2+}$  has spin  $S = 5/2$ , but, due to the large and positive single-ion anisotropy  $\propto (S^z)^2$  at the quantum well interface, the higher spin components freeze out in the sub-Kelvin range, leaving behind a spin-1/2 doublet [16]. Moreover, the single-ion anisotropy implies that the Kondo interaction with this effective spin-1/2 impurity is anisotropic, with  $J_x = J_y = 3J_z = 3J_I$ , where  $J_I > 0$  is the isotropic bulk spin-exchange coupling [17]. Its value can be assessed from the sp-d exchange integrals for the bulk conduction electrons in  $\text{Hg}_{1-x}\text{Mn}_x\text{Te}$  [32]. With the  $\text{Mn}^{2+}$  ion located within the penetration depth  $\kappa$  from the edge, a rough estimate yields an expected value of  $J_I/a_0 \approx 10$  meV. Turning to the Rashba coupling  $\alpha$ , gate controls have been demonstrated in the laboratory with  $\hbar\alpha$  for an HgTe quantum well device running from  $5 \times 10^{-11}$  eVm to  $1 \times 10^{-10}$  eVm as the bias of a top gate is varied from 2 V to -2 V [33]. As for the value of the interaction parameter  $K_0$  in an HgTe quantum well, estimates range between 0.5 and 1 [8, 14, 34–36], and depend on the geometry and composition of the heterostructure. Collecting the numbers, and putting  $a_0 \approx 0.5$  nm [37],  $v_F \approx 5.0 \times 10^5$  m/s [1],  $D \approx 300$  meV [1], we can use Eq. (13) to numerically plot the  $\delta I$ - $V$  characteristics for different values of  $\alpha$  and  $K_0$  [38], here choosing  $T = 30$  mK ( $> T_K$ ), see Fig. (2). As revealed by the graphs, the Rashba-dependence of  $\delta I$  is appreciable, and should allow for an experimental test [39].

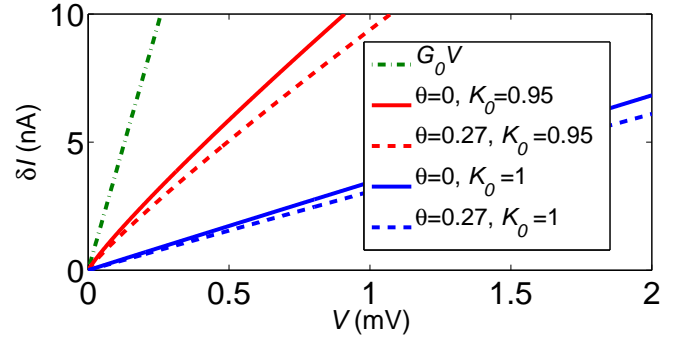


FIG. 2: The RG-improved current correction (13) at  $T = 30$  mK as a function of applied voltage, for different values of  $K_0$  and  $\theta$ . The dashed lines represent  $\theta \approx 0.27$ , corresponding to  $\hbar\alpha = 10^{-10}$  eVm. Other parameters are defined in the text. The QSH edge current  $G_0 V$  is plotted as a reference.

*Concluding remarks.* We have studied the combined effect of a Kondo and a Rashba interaction at the edge of a quantum spin Hall system. The interplay between an anisotropic Kondo exchange and the Rashba interaction is found to result in a non-collinear electron-impurity spin interaction. A perturbative RG analysis indicates that



this interaction may block the Kondo effect when the electron-electron interaction is weakly screened. We conjecture that this surprising result – challenging a time-honored expectation that the Kondo effect is blind to time-reversal invariant perturbations [15] – is due to the breakdown of single-particle physics in one dimension. It remains a challenge to unravel the microscopic scenario behind this intriguing phenomenon. In the second part of our work we derived expressions showing how charge transport at the edge is influenced by the simultaneous presence of a magnetic impurity and a Rashba interaction. A case study suggests that the predicted current-voltage characteristics should indeed be accessible in an experiment. Most interestingly, its manifest dependence on the gate-controllable Rashba coupling breaks a new path for charge control in a helical electron system.

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